In: Ubuz, B.; Haser, Ç.; Mariotti, M. A. (Eds.): Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education. Ankara, Turkey: Middle East Technical University, 2013, pp. 572–573

IMPROVING THE COVARIATIONAL THINKING ABILITY OF SECONDARY SCHOOL STUDENTS

Tobias Rolfes, Jürgen Roth, & Wolfgang Schnotz

University of Koblenz-Landau, Campus Landau, Germany

Keywords: Functions, Covariation, Forms of Representation

THEORETICAL BACKGROUND

Vollrath (1989) considers three aspects of the concept of function to be essential. First, in the point-wise view, every value of the domain corresponds to exactly one value of the range. Second, in the dynamic view, the aspect of covariation requires taking into account the neighborhood of the point: How does the function value vary if the input value is varied? Third, the global view contemplates the function as a whole. This is necessary if statements are made about symmetry, for example.

Researchers point out that the aspect of covariation is not sufficiently implemented in mathematics curricula (Malle, 2000; Thompson, 1994). Instead, school teaching mainly focuses on the point-wise aspect of functions even though it is hardly possible to construct adequate mental models of the concept of function without considering the covariational aspect. Despite researchers' consensus on the importance of this aspect, there is a lack of research on how to develop it in secondary education.

AIMS AND RESEARCH QUESTIONS

By developing empirically based training sessions in covariational thinking, we hope to better prepare students for the infinitesimal calculus and also to show a way of improving a neglected aspect of mathematical literacy in secondary grades. A research hypothesis is that covariational thinking is not restricted to upper secondary grades and that students from grade 5 onwards are able to make substantial improvements with regard to this aspect of functional thinking. On the one hand, we want to identify students' misconceptions when dealing with covariational tasks. On the other hand, we aim at identifying students' cognitive resources in the area of covariational thinking. As we conjecture that the forms of representation play a decisive role, we want to find out which form of representation is more intuitive when dealing with covariational tasks.

METHOD

A training session in seventh grade ($n = 27$) started with a material-based analysis of the covariation of different functional dependencies (linear vs. quadratic) with a discrete domain.Initially , the students explored the covariation in a qualitative and quantitative manner based on self-generated representations, namely tables of values and graphs, in a discovery learning environment for 20 minutes. Afterwards, the solutions were discussed in a teacher-class dialog for another 20 minutes. At the end of the lesson, the students were given a paper-and-pencil test consisting of six tasks, which were analyzed with mixed methods.

RESULTS AND PRACTICAL IMPLICATIONS

In the analysis of misconceptions we discovered that students confused the first and second difference. Many students incorrectly identified a function with constant second differences as a linear function instead of a quadratic function. This conflation forms an essential obstacle when dealing with covariation. Research on the development of teaching concepts to deal with this problem is still needed.

Furthermore, some students extrapolated linear growth as if it were proportional growth. These findings are consistent with the documented preference for proportional reasoning (De Bock, Van Dooren, Janssens, & Verschaffel, 2002). As a consequence, teaching in early secondary grades has to put more emphasis on non-proportional growth (e.g., exponential, quadratic, or logistic growth) to prevent an overgeneralization of proportional reasoning.

In a quantitative analysis, the role of the form of representation was investigated in a within-subjects design. The students performed significantly better at value table production than graph construction (sign test: $g = .35$, $p < .001$). This raises the question of whether the table of values activates students' cognitive resources in covariational thinking more than graphs. Accordingly, further research on this topic is necessary.

Moreover, students had more difficulty extrapolating quadratic functions than linear functions (sign test: $g = .24$, $p < .01$). This result underlines that school teaching should deal with different types of growth as early as possible.

REFERENCES

De Bock, D., Van Dooren, W., Janssens, D., & Verschaffel, L. (2002). Improper use of linear reasoning: An in-depth study of the nature and the irresistibility of secondary school students' errors. *Educational Studies in Mathematics, 50* (3), 311–334.

Malle, G. (2000). Zwei Aspekte von Funktionen: Zuordnung und Kovariation [Two Aspects of Functions: Assignment and Covariation]. *Mathematik lehren*, (103), 8–11.

Thompson, P. W. (1994). Students, Functions, and the Undergraduate Curriculum. In E. Dubinsky, A. H. Schoenfeld, & J. J. Kaput (Eds.), *Research in Collegiate Mathematics Education I* (pp. 21–44). Providence, RI: American Mathematical Society.

Vollrath, H.-J. (1989). Funktionales Denken [Functional Thinking]. *Journal für Mathematik-Didaktik*, 10(1), 3–37.