Inspiring teaching and learning of functional thinking by experiments with real and digital materials
Inspiring teaching and learning of functional thinking

1 Inspiring teaching and learning by teaching-learning-laboratories

2 Inspiring learning: Experiments with real or digital materials

3 Inspiring teaching: Video vignettes for the analysis of teaching and learning processes
Jürgen Roth • Inspiring teaching and learning of functional thinking by experiments with real and digital materials

Teaching-Learning-Laboratories

Duisburg Essen
University of Duisburg-Essen

Koblenz Landau
University of Koblenz-Landau

students lab
lab for pre- and in-service teachers
research laboratory
mathe-labor.de
zentral.uni-koblenz-landau.de
Centre for Education and Research at Teaching-Learning-Laboratories

Teaching-Learning-Laboratory
Inspiring teaching and learning of functional thinking

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2  Inspiring learning: Experiments with real or digital materials

3  Inspiring teaching: Video vignettes for the Analysis of teaching and learning processes
Selection of situations
- practicability
- comparable actions
- use media advantages
- different functional relationships

real materials ↔ digital materials

Lichti & Roth 2018

www.geogebra.org/m/ncqCZurb
**Task design**

- **Tasks**
  - Determine values (measurement)
  - Create tables of values
  - Working with graphs
    - “real”-group: draw graphs
    - “digital”-group: observe genesis of graphs
    - both groups: additional drawing of and working with graphs
  - Interpolation, application and transfer

- **Forms of representation**
  - situation → graph
  - verbal description → table
  - situation → verbal description
  - graph → table
  - situation → table
  - verbal description → graph
  - situation → graph

Lichti & Roth (2018, 2019)
Intervention study

**Experimental group 1**
- Pretest: 40 min
- One week
- Intervention (4 h): Real materials
- Posttest: 40 min

**Experimental group 2**
- Pretest: 40 min
- One week
- Intervention (4 h): Digital materials
- Posttest: 40 min

**Control group**
- Pretest: 40 min
- One week
- No intervention
- Posttest: 40 min

- Thirteen classes of 6th-graders ($N = 282$)
- School year 2015/16 just before summer holidays

Performance in functional thinking

digital materials group, $N = 123$
real materials group, $N = 111$

Average performance in functional thinking in logit

![Graph showing performance differences between digital and real materials groups pretest and posttest with statistical analysis results.](image)

- Pretest:
  - Digital materials group: $p < 0.001$, $d = 1.40$
  - Real materials group: $p < 0.001$, $d = 0.85$

- Posttest:
  - Digital materials group: $0.41$
  - Real materials group: $0.09$

Qualitative content analysis
Tasks from intervention and posttest with open response formats

Reason?

Lichti & Roth (2018, 2019)

Jürgen Roth • Inspiring teaching and learning of functional thinking by experiments with real and digital materials
Here you can see various vessels and filling graphs. Match them and justify your choice.

<table>
<thead>
<tr>
<th>Category</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>shape of the vessel</td>
<td>The glass is getting wider.</td>
</tr>
<tr>
<td>course of the graph</td>
<td>Because the graph is pretty flat.</td>
</tr>
<tr>
<td>rise of the water</td>
<td>First the water rises slower, then faster.</td>
</tr>
<tr>
<td>state</td>
<td>The graph is steep. The bowl is flat.</td>
</tr>
<tr>
<td>variation</td>
<td>The graph becomes steeper and steeper.</td>
</tr>
</tbody>
</table>

Lichti & Roth (2018)
Filling vessels: results

- **Real materials group**
  Students argue significantly more frequently
  - with the shape of the vessel
    \(\chi^2 = 14.79, df = 1, p < 0.001, V = 0.15\)
  - with states
    \(\chi^2 = 4.361, df = 1, p = 0.037^*, V = 0.08\)

- **Digital materials group**
  Students argue significantly more frequently
  - with the course of the graph
    \(\chi^2 = 6.62, df = 1, p = 0.01^*, V = 0.10\)
  - with variations
    \(\chi^2 = 6.955, df = 1, p = 0.008^{**}, V = 0.11\)

Lichti & Roth (2018)
This graph shows how a racing cars’ speed changes during the second lap of a racing track that is 3 km long and flat.

Which racing track did the car drive on? The pictures are bird’s eye views of the tracks.

Justify your choice.

- A
- B
- C
- D
- E

Lichti & Roth (2018)

<table>
<thead>
<tr>
<th>Category</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>speed and curves</td>
<td>A car must slow down in curves.</td>
</tr>
<tr>
<td>different curves</td>
<td>There are three curves: two flat and one steep.</td>
</tr>
<tr>
<td>graph-as-picture error</td>
<td>Because the track looks like the graph.</td>
</tr>
<tr>
<td>Incorrect interpretation of the racing tracks</td>
<td>The racetrack [A] has three curves.</td>
</tr>
</tbody>
</table>
Racing cars: results

- **Essential for the solution**
  - Graph $\Rightarrow$ three different curves

- **Real materials group**
  - Students are significantly more frequently **not** able to apply their knowledge of the relationship between speed and curves. ($\chi^2 = 6.304$, $df = 1$, $p = 0.012^*$, $V = 0.21$)
  - graph-as-picture error: 36%

- **Digital materials group**
  - graph-as-picture error: 0%

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Lichti & Roth (2018)
Results at a glance

- **Quantitative analysis**
  - Development of functional thinking: digital > real

- **Qualitative analysis**
  - Argumentation with graphs: digital > real
  - Argumentation with real situations: digital < real
  - Argumentation with covariation: digital > real
  - Argumentation with mapping: digital < real
  - Qualitative approach to a graph (interpreting and/or drawing): digital > real
  - Single pairs of values (interpreting and/or drawing): digital < real

Lichti & Roth (2018)
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Video vignettes for the Analysis of teaching and learning processes

Bartel & Roth (2017)

http://vivian.uni-landau.de

Learning Environment: Topic and Goals

Student level
- Task
- Materials
- Student documents

Meta level
- Student profile
- Temporal classification
- Diagnostic tasks

Student profile
- S1
- S2
- S3
- S4

Temporal classification

Diagnostic tasks

00:51 02:52
Video vignettes for the Analysis of teaching and learning processes

Bartel & Roth (2017a)

Vignette 1
Vignette 2
Vignette 3
Vignette 4
Vignette 5
Vignette 6

Learning Environment: Topic and Goals

Meta level

Student profile
Temporal classification
Diagnostic tasks

Functions
Terms
Fractions
Area and volume
Logout


http://vivian.uni-landau.de
Video vignettes for the Analysis of teaching and learning processes

Bartel & Roth (2017a)

Learning Environment: Topic and Goals

Diagnostic tasks: Video vignette 1

2. Describe the problems that occur in the videotaped situation.
Diagnostic tasks: Video vignette 1

2. Describe the problems that occur in the videotaped situation.
Peter, Paul and Mary go to school and live in the same street. At the end of the street is their school. Every morning they walk to school. The picture shows where the three were yesterday at different times.

Write a story about Peter's way to school yesterday.
Diagnostic tasks: Video vignette 1

2. Describe the problems that occur in the videotaped situation.

Student profile:
- Peter
- Paul
- Mary
- Time
- Distance to school

Student documents:

Student 1:

Eines Tages ging Peter ganz alleine zur Schule.
Er schloss die Türe von sich her.
Peter geht zur Schule und muss an eine Ampel warten, dann geht er weiter und muss einen Weg wegen einer Baustelle machen. Dan steht er kurz und wird von einer Ampel zum Ende Schloss
beilt er sich da er nicht zu spät kommt.

Student 2:

In some cases, the students fail to grasp the relations represented in the graph (e.g. axis labeling).
Diagnostic tasks: Video vignette 1

2. Describe the problems that occur in the videotaped situation.

You replied:

In some cases, the students fail to grasp the relations represented in the graph (e.g. axis labeling).

Experts have given the following answers:

- At first, not all pupils correctly recognize the context depicted. At times, the "speed" is considered instead of the...
Diagnostic tasks

- Work through student tasks.
- Describe observations.
- Interpret observations and give reasons for interpretations (basic ideas, student (mis)concepts, ...)
- Suggest and justify teaching activities.

Diagnostic tasks: Video vignette 1

2. Describe the problems that occur in the videotaped situation.

You replied:

*In some cases, the students fail to grasp the relations represented in the graph (e.g. axis labeling).*

Experts have given the following answers:

- At first, not all pupils correctly recognize the context depicted. At times, the "speed" is considered instead of the "distance from school".
- A negative gradient of the graph is interpreted as a higher speed than a positive gradient (although the sign of the gradient at this point has no influence on the speed).
- The students do not understand what the horizontal sections of the graph mean. They assume that there is still distance covered in this section, but the speed does not change.
- It is not recognized where the school is in the coordinate system.
- The graph-as-picture error occurs because it is argued that as the graph rises, the person goes up a mountain and thus slows down.
- Uncertainty as to which concept of time is used in the task: The time the students still need until they reach school or the "normal" time that goes on while something is happening.
Inspiring Teaching and Learning

Lab for pre- and in-service teachers

Math is more

Teaching-learning-laboratory

Learning environment

Video vignettes for the Analysis of teaching and learning processes

Student level

Learning Environment: Topic and Goals

Meta level

Student profile

Materials

Student documents

S1 S2 S3 S4

Temporal classification

Diagnostic tasks

www.vivian.uni-landau.de

www.mathe-labor.de

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Thank you for your attention.