In: Dooley, T., & Gueudet, G. (Eds.). (2017). Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education (CERME10, February 1-5, 2017). Dublin, Ireland: DCU Institute of Education and ERME.

Real materials or simulations?

Searching for a way to foster functional thinking

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Keywords: Real materials, simulation, functions, functional thinking.

Relevance

Living in a world strongly influenced by intelligent technology, it is indispensable to know in which contexts this technology can be beneficial and in which contexts the 'real world' should be used for teaching mathematics. Considering the topic of functional relationships, the need to foster pupils' functional thinking (FT) from the very first beginning arises. Even though the topic is important for mathematics education in every grade, pupils show a lot of misconceptions (Leinhardt et al., 1990). Therefore and because of our multimedia life, we need to ask, if FT should be fostered with computer-simulations (GeoGebra) or real materials (like cubes, pencils…).

Theoretical background

FT consists of three fundamental aspects: mapping, covariation, and function as object (Vollrath, 1989). Previous research shows that the use of real materials as well as the application of computer-simulations can lead to a learning progress in this complex topic. On the one hand, real materials make it possible to experience functional relationships physically (Ludwig & Oldenburg, 2007). Learning in such a setting has long-lasting effects, i.e. pupils can recall results and working methods better (Vollrath, 1978). On the other hand, simulations enable pupils to explore functions in different ways. Pupils can vary variables systematically and use the multiple-representation system (Balacheff & Kaput, 1997). Thus, e.g. covariation gets perceptible. Summing up, simulations become a mediator between pupils and mathematical phenomena (Hoyles & Noss, 2003).

Methodology

After constructing a test to measure FT we derived topics that can be used to foster FT with real materials and simulations from theory: the relationship between volume and fill height of vessels, edge length and volume of a cube, diameter and circumference of discs, rotation-number while sharping a pencil and its remaining length.

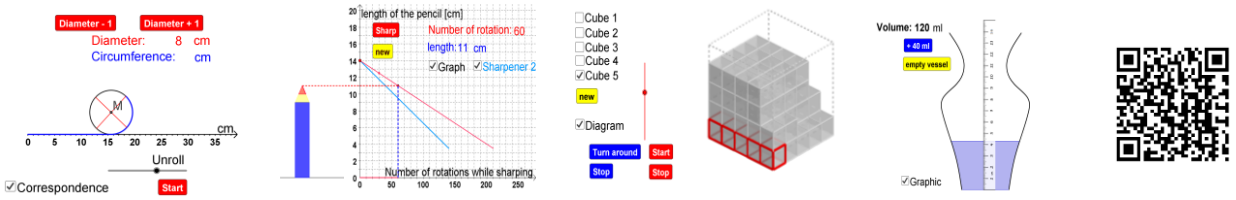

Figure 1 Interfaces of the used simulations done with GeoGebra

Then we designed an intervention-study (pre-post-control-group-design, randomized experimental-groups) that was implemented in grade 6 (age 11-12, N = 282). During the intervention (4 lessons) pupils had to work on learning-tasks individually to foster their FT. They were not instructed or

supported by a teacher. While part of them were using real materials (experimental-group 1, N = 111) the others worked with computer-simulations (experimental-group 2, N = 123). The learning-tasks in both groups were equivalent, only the medium differed. The control-group (N = 49) worked on pre- and posttest, only. Data analysis was done with item-response-theory (IRT). First, we estimated item difficulties by use of virtual persons. Then we did a 2-dim. Rasch-model (dim. 1: pretest, dim. 2: posttest) with fixed item difficulties to estimate the person ability FT. Finally, a mixed ANOVA (Field et al., 2013) using 10 sets of plausible values to compare pupils' FT in pre- and posttest was conducted.

Results

The mixed ANOVA (between: intervention, within: time) leads to the result, that there is a significant main effect of time ($F(1, 22.71) = 68.16$, $p < .001$, $\eta^2 = .089$) and also a significant interaction effect of time and intervention ($F(1, 23.69) = 7.65$, $p = .003$**, $\eta^2 = .044$) . A pairwise t-test showed that real materials as well as computer-simulations lead to a significant increase of FT (real materials: $p < .001$, Cohen's $d = .49$, computer-simulations: $p < .001$, Cohen's $d = .83$). In contrast, the control groups' FT increases not significantly ($p = 1$, Cohen's $d = .22$). Thus, it needs to be concluded that FT can be fostered by use of real materials as well as computer-simulations. Nevertheless, effect sizes show that computer-simulations should be the method of choice for fostering FT.

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