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Assessment of students' thinking when working with graphs of functions – Promoting pre-service teachers' diagnostic competence

Rita Hofmann[1] and Jürgen Roth[2]

[1]University of Koblenz-Landau, Campus Landau, Germany; hofmannr@uni-landau.de

[2] University of Koblenz-Landau, Campus Landau, Germany; roth@uni-landau.de

Diagnostic competences are essential for teacher actions; however pre-service teachers often do not have the opportunity to train these skills at university. Thus, there is a need to find out the best way to promote diagnostic competences in teacher training. During the last decades, several projects introduced videos as a tool for the training of diagnostic skills, but there is no evidence that pre-service teachers really acquire diagnostic skills better by analysing videos than by analysing tasks. The present study contributes to this growing area of research by exploring which one of these two methods promotes diagnostic skills better. Video analysis and task analysis are compared as training methods in an intervention study with a pretest-posttest design. Fostering preservice teachers' diagnostic skills with focus on students' abilities, problems and misconceptions with graphs of functions, is the specific objective of our study.

Keywords: Pre-service teacher training, diagnostic competence, formative assessment, graphs of functions, video vignettes.

Theoretical background

Good lessons require a lot of competencies on the teacher's side. Diagnostic skills, in particular, are an important part of teachers' professional knowledge and competence (e.g. Baumert & Kunter, 2006). Weinert (2000) regards diagnostic competence as one out of four basic and essential competences of teachers. Having good diagnostic skills enables a teacher to differentiate and individualise amongst learners – an ability that becomes increasingly important in today´s classrooms, "[…] because lessons can no longer be planned completely in advance, and teachers have to make many decisions in the midst of instruction about how to proceed" (van Es & Sherin, 2002, p. 574). Good lessons require teacher actions that are adapted to the students' needs and abilities and, therefore, are based on diagnostic information (Klug et al., 2013; Schrader & Helmke, 2001). For the adaptation of teacher actions to pupils' needs during a lesson, relevant information needs to be obtained during the students' whole learning process. Getting an insight into the students´ abilities only through the results of a final exam, is often too late to work on the students' problems and misconceptions. For this reason, our focus lies on diagnostics which take place in the learning process of the students where the teacher is still able to guide and influence the learners and their learning process. In the following, an overview of different aspects concerning diagnostics and assessment will be given.

Diagnostic competence

The term "diagnostic competence" is often used in the literature, but there is no agreement on a definition of this expression. A wide-spread definition would be that diagnostic competence involves all the abilities of an evaluator enabling him to correctly asses other people (Schrader, 2010). Artelt and Gräsel (2009) understand diagnostic competence as the teachers' competence to

evaluate the characteristic traits of their students in an adequate way and to suitably assess the demands of learning and of the tasks. Except of these two, various other definitions are used. Diagnostic competence is often described as "accuracy" in teachers' judgements – mostly in correlation with standardized tests – and therefore concerns the students' achievement in tests (Klug et al., 2013). Other definitions refer to the learning process of the students itself. In this regard Weinert (2000) defines diagnostic competences as:

> […] an amount of abilities to continuously asses during lesson the state of knowledge, the learning progresses and the performance issues of the individual students as well as the difficulties of different learning-tasks, so that the teaching actions can be based on diagnostic insights. (Weinert, 2000, p.16, own translation)

All of these definitions have in common that diagnostic skills are presented as the tool allowing the teachers to gain information about the learners. This information can be used for different pedagogical decisions like grading and lesson planning (makroadaptations), but also for short-termed interventions during lessons (mikroadaptations) (Schrader, 2013). As our study does not focus on achievements in tests but on the learning process of the students, we refer to the definition of Weinert. Moreover, there are different facets of diagnostic competences (e.g. Praetorius, Lipowsky, & Karst, 2012), so that we prefer to use the term diagnostic skills, as we focus on specific parts of it: the analysis of tasks and the analysis of video sequences – both with regard to abilities, problems and misconceptions of students working on tasks with the content functional relationships.

Formative assessment vs. summative assessment

The terms formative and summative assessment are quite similar to the foregoing described diagnostic competence. Again, there is no common and widely accepted definition although they are widespread in the international literature (Black & Wiliam, 1998). While summative assessment corresponds to the evaluation of students' academic achievements, formative assessment can be equated to diagnoses during learning processes. According to Bell and Cowie (2001b, p. 538), such diagnoses during learning processes "could include continuous summative assessment", which is why the authors "explored formative assessment as classroom assessment to improve learning (and teaching) during the learning". Bell and Cowie (ibid.) distinguish between planned formative assessment and interactive formative assessment. The former describes an assessment activity which is planned in advance, the latter includes assessments that arise out of learning activities during the lesson (Bell & Cowie, 2001a). The purpose of interactive formative assessment is to help the students by accompanying the learning process (Bell & Cowie, 2001a). According to Bell and Cowie (2001a, p. 86) this process involves three parts: noticing, recognizing and responding. Noticing in this context means to gather information about the patterns of thought and actions of the students. This information is gathered while the pupils are working or talking. Thus, this interpretation differs from the term "noticing" described by van Es and Sherin (2002). In contrast to the meaning of "noticing" characterized by van Es and Sherin, which already includes the identification of important aspects of a teaching scenario, Bell and Cowie (2001a) regard the recognition of relevant interactions and moments as a second step. "Recognising may be differentiated from noticing in that it is possible to observe and note what a student does without appreciating its significance" (Bell & Cowie, 2001a, p. 88). Consideration of "responding" as one of

the stages of interactive formative assessment shows, that the noticing or assessment should not stand alone – the following action of the teacher is indispensable.

In this sense, formative assessment involves diagnostic as well as didactical competencies - action competence, respectively. Diagnosis/ noticing and the action which follows up the diagnosis are both parts of formative assessment. To sum up, the subject of our study is diagnostic competence according to Weinert (2000) and the following teacher action. Thus, the regarded skills manifest themselves in the three stages of interactive formative assessment: noticing, recognizing, and responding (Bell and Cowie, 2001a).

Graphs of functions

The focus of the diagnosis in our project is on the students' learning processes while working with graphs of functions. The interpretation and construction of graphs of functions are essential skills - not only in mathematics education. The ability to use different (external) representations is an important issue here. It is one of the six mathematical competences mentioned in the German educational standards for mathematics and also influences two of the remaining standards (KMK, 2004). Moreover, the use of graphs of functions is essential for the topic "functional relationships", being one of five central topics of mathematics education (KMK, 2004). In addition to that, the abundance of graphs of functions in our everyday life (e.g. functional relationships or graphical representations of data) makes them indispensable in teaching and learning. Nevertheless, previous research has shown that dealing with graphs of functions can be difficult and easily leads to misconceptions. In the literature one can find a lot of those mistakes and misconceptions (e.g. Nitsch, 2015; Leinhardt et al., 1990; Clement, 1985; Bell & Janvier, 1981), like the graph-as-picture misconception, the slope-height confusion or the interval-point confusion. Moreover, what the students think a function is or how a graph of a function should look like (concept image) does not always correspond to the definition of a function, the students have in mind (concept definition) (Tall & Vinner, 1981).

However, not all of these mistakes and misconceptions are visible on the surface but they need to be uncovered in time. Otherwise, there is the danger of a consolidation of wrong thinking making it very hard to work against them (Nitsch, 2015). In this case, wrong conceptions might still be present when students leave school or even when they enrol at university. Teachers need to be able to diagnose students´ misconceptions and difficulties in time in order to foster their correct use of graphs of functions.

Giving effective feedback is a crucial aspect of teacher-learner interactions (Hattie, 20120), but often there is a lack of time for reflection and decisions on necessary actions to be taken (Black & William, 2009). The perception and processing of crucial situations often takes place intuitively – "on the fly" – when the teacher is monitoring the classroom and listening to student conversations while students are working with their partners or in groups. This is a highly demanding situation for teachers (William & Thompson, 2007). Consequently, in the beginning of teaching, teachers can experience an overloading by the wealth of information. Thus, the skills to notice, recognize and respond should already be fostered during preservice teacher training. A common way to train diagnostic skills is the analysis of tasks as it can easily be embedded in university teacher training. Thereby the university students reflect the skills which are needed to solve a task as well as

problems which can occur with the task. This method focuses on skills which are primarily necessary in lesson planning. No influence of task analysis on teachers´ diagnostic skills could be found yet. It could be assumed, that a good analysis of tasks helps a person to notice things – which are expected through the analysis – in reality. Nonetheless the analysis of gestures is not part of this method and can still be a difficulty for beginning teachers. Furthermore, noticing in a situation is more complex and can be cognitive overwhelming. Therefore, another approach to train such diagnostic skills is the use of videos as part of the training of diagnostic competences, as videos are very close to reality (compare Janík et al., 2009).

Up to now, several studies have shown that pre-service teachers often do not have the opportunity to train their diagnostic skills so that these competences are only poorly developed (Ostermann et al., 2015; Praetorius, Lipowsky, & Karst, 2012). For this reason, we want to foster these skills already during the university teacher training.

Research Question

The goal of our research is to enhance pre-service teachers´ diagnostic skills through experimental settings at university. As mentioned before, there are different aspects of diagnostic skills, all important for professional teaching. On the one hand, a teacher should be able to identify possible difficulties of a task and be aware of the skills needed for solving the task. On the other hand, the teacher needs to be able to identify the concrete difficulties and misconceptions an individual student has and to react appropriately. The analysing of tasks is one common way to train diagnostic skills of pre-service teachers. During the last decades videos were introduced as training tool for diagnostic skills as well. Looking at the two approaches to the training of diagnostic skills, several questions arise that need to be answered:

1. How does the training of task analysis influence the skills for analysing learning situations?
2. How does the training of analysing videos influence task-analytical skills?

Furthermore, as the diagnoses should be the basis for teacher action, the impact of both trainings with regard to this issue is another interesting part of the investigation:

3. Which intervention results in a noticeable improvement of the actions following the diagnoses?

Method

In order to verify the effects of the different trainings on the preservice teachers' diagnostic skills we will conduct an intervention study using a pre-posttest design. Thus, it will be a setting with two experimental groups: Experimental group one (EG1) will practice diagnostic skills by analysing videos, experimental group two (EG2) by analysing the tasks the students work on (Figure 1). The preliminary study will be conducted in winter term 2016/2017. The participating pre-service mathematic teachers (approximately 60 persons) are currently attending the same lecture in mathematics education (didactic of algebra) and will be randomly distributed into the two experimental groups. The participants of both groups receive the same content input during the lecture. The information given in the lecture will be on functional relationships and particularly focus on the representation graph of functions. Furthermore, the skills which the learners shall

acquire as well as possible student mistakes and misconceptions which can occur during learning, are of special interest.

In the intervention the participants of EG1 are asked to analyse video-vignettes. The participants of EG2 have to analyse tasks which contain the construction or the interpretation of graphs of functions. The focus of both analyses lies on diagnosis of errors regarding problems and misconceptions as well as the skills the students already have or need. The video-sequences used for the intervention can be watched multiple times, stopped at any point and the participant can jump to any point in the video that is of interest to him. This circumstance is meant to help the pre-service teachers as well as possible while they are analysing the learning process of the pupils. The tasks which are given to the participants of EG2 are the same tasks used for the video vignettes. Therefore, differences between experimental groups are limited to the characteristics of the learning resources. The analysis – both of the videos-vignettes and the tasks – happens at each individuals' home, not during the university lecture. In contrast to the test situations, there will be no time constraint during the intervention in order to foster the development of diagnostic skills.

During the pretest, additional data will be collected: teaching experience, attended university lectures in education (other subjects included), differentiating between those already attended, and those happening in the meantime of the intervention. Knowing these influences gives us the opportunity to consider them as covariates for the computation and the results.

The pre- and posttest will be conducted to measure the diagnostic skills of the pre-service teachers at the beginning and the end to be able to see the changes of these skills between before and after the intervention. The tests inquires diagnostic skills which are important for the preparation of lessons as well as those needed to be able to notice situations relevant for successful learning in class. Furthermore reactions based on the participants' diagnoses will be part of the inquiry. The test for diagnostic competencies asks participants to first analyse tasks. Then, a three-minute video will be presented, showing pupils working on the tasks previously analysed. The video can only be watched once and doesn't provide the possibility to pause. This way, we are trying to create a test-situation which is as close to reality as possible. The test includes both open and closed questions asking the participant to communicate what they have noticed and to reason about their findings.

By testing both types of analysis, we want to investigate whether different diagnostic skills have influences on each other. Moreover we expect the test to resolve, whether one method is superior to the other one. This would be the case if for one training method superior gains in both types of diagnostic skills could be observed.

Both settings of the intervention and the tests for diagnostic competence are embedded in the learning environment ViviAn (see Figure 2) developed by Bartel and Roth (2015). This learning environment provides a combination of video vignettes and further material and thereby further approximates the information available in real-life teaching situations. Hence, the user gets information about the students (type of school, grade, sex), the content and the learning goals of the entire lesson, and the materials the students use such as the given task and the materials (for example a big sheet of paper with a graph of a function on it). The students' protocols (products) are only available to the participants of EG1 who are analysing videos. As the participants of EG2 analyse the task in more general they shall not be influenced by the solution of the pupils.

Figure 2: The learning environment ViviAn (Bartel & Roth, 2015)

The data will be analysed with mixed methods. The approach of qualitative content analysis (Mayring, 2008) will be applied to create a coding guideline. Thereby the answers of the participants will be compared to experts' diagnoses. As experts serve mathematics teachers and academic staff working in the field of didactics of mathematics. These experts' diagnoses will be used as a criterion norm for the measurement of diagnostic skills by using the resulting criteria to rate the participants' answers. To resolve group differences descriptive statistics as well as inferential statistics with variance analysis will be considered.

Expected results

The preliminary study was conducted in winter term 2016/2017. It will reveal potential problems concerning our approach, the used material and tasks. Based on these findings we will be able to improve our approach and the used material. Moreover, the preliminary study contributes to the investigation of differences between the diagnoses of tasks and videos. Prospectively, with the results of the main study, we will then be able to point out, whether trainee teachers better acquire diagnostic skills by analysing videos than by analysing tasks. Furthermore it will provide insight into whether different aspects of diagnostic skills have an influence on each other.

References

Artelt, C., & Gräsel, C. (2009). Diagnostische Kompetenz von Lehrkräften [Diagnostic competence of teachers]. *Zeitschrift für Pädagogische Psychologie*, *23*(34), 157–160. doi:10.1024/1010-0652.23.34.157

Bartel, M.-E., & Roth, J. (2015). Diagnostische Kompetenz durch Videovignetten fördern [Promoting diagnostic competence through video vignettes]. In F. Caluori, H. Linneweber-Lammerskitten, & C. Streit (Eds.), *Beiträge zum Mathematikunterricht 2015* (pp. 1033–1036). Münster: WTM.

Baumert, J., & Kunter, M. (2006). Stichwort: Professionelle Kompetenz von Lehrkräften [Keyword. Professional competence of teachers]. *Zeitschrift für Erziehungswissenschaft*, *9*(4), 469–520. doi:10.1007/s11618-006-0165-2

Bell, A., & Janvier, C. (1981). The interpretation of graphs representing situations. *For the Learning of Mathematics, 2*(1), 34–42.

Bell, B., & Cowie, B. (2001a). *Formative assessment and science education*. *Science & technology education library: Vol. 12*. Dordrecht, Boston: Kluwer Academic.

Bell, B., & Cowie, B. (2001b). The characteristics of formative assessment in science education. *Science Education, 85*(5), 536–553. doi:10.1002/sce.1022

Black, P., & Wiliam, D. (1998). Assessment and Classroom Learning. *Assessment in Education: Principles, Policy & Practice, 5*(1), 7–74. doi:10.1080/0969595980050102

Black, P., & Wiliam, D. (2009). Developing the theory of formative assessment. *Educational Assessment, Evaluation and Accountability, 21*(1), 5–31. doi:10.1007/s11092-008-9068-5

Clement, J. (1985). Misconceptions in graphing. In L. Streefland (Ed.), *Proceedings of the Ninth International Conference for the Psychology of Mathematics Education* (pp. 369–375). Utrecht,

Hattie, J. (2012). *Visible learning for teachers. Maximizing impact on learning*. London: Routledge.

Janik, T., Janikova, M., Knecht, P., Kubiatko, M., Najvar Petr, Najvarová, V., & Sebestová, S. (2009). Exploring Different Ways of Using Video in Teacher Education: Examples from CPV Video Web. In T. Janik & T. Seidel (Eds.), *The Power of Video Studies in Investigating Teaching and Learning in the Classroom* (pp. 207–224). Münster: Waxmann.

Klug, J., Bruder, S., Kelava, A., Spiel, C., & Schmitz, B. (2013). Diagnostic competence of teachers: A process model that accounts for diagnosing learning behavior tested by means of a case scenario. *Teaching and Teacher Education, 30*, 38–46. doi:10.1016/j.tate.2012.10.004

KMK. (2004). *Bildungsstandards im Fach Mathematik für den Mittleren Schulabschluss*. Beschluss vom 4.12.2003 [Educational standards in mathematics for middle graduation. Resolution of 4th December 2003].

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, Graphs, and Graphing: Tasks, Learning, and Teaching. *Review of Educational Research, 60*(1), 1–64. doi:10.3102/00346543060001001

Maier, U. (2010). Formative Assessment – Ein erfolgversprechendes Konzept zur Reform von Unterricht und Leistungsmessung? [Formative Assessment – A promising concept to the reform of lessons and assessment?] *Zeitschrift für Erziehungswissenschaft, 13*(2), 293–308. doi:10.1007/s11618-010-0124-9

Mayring, P. (2008). *Qualitative Inhaltsanalyse. Grundlagen und Techniken*. [Qualitative content analysis. Basics and techniques]Weinheim: Beltz.

Nitsch, R. (2015). *Diagnose von Lernschwierigkeiten im Bereich funktionaler Zusammenhänge: Eine Studie zu typischen Fehlermustern bei Darstellungswechseln* [Diagnosis of learning issues in the field of functional relationships. A study on typical error pattern in the change of representations]. Wiesbaden: Springer Spektrum.

Ostermann, A., Leuders, T., & Nückles, M. (2015). Wissen, was Schülerinnen und Schülern schwer fällt. Welche Faktoren beeinflussen die Schwierigkeitseinschätzung von Mathematikaufgaben?

[Knowing what is difficult for the students. Which factors influence the assessment of the difficulty of mathematic tasks?] *Journal für Mathematik-Didaktik, 36*(1), 45–76. doi:10.1007/s13138-015-0073-1

Praetorius, A.-K., Lipowsky, F., & Karst, K. (2012). Diagnostische Kompetenz von Lehrkräften: Aktueller Forschungsstand, unterrichtspraktische Umsetzbarkeit und Bedeutung für den Unterricht [Diagnositic competence of teachers: state-of-art in research, implementation and importance for teaching]. In R. Lazarides & A. Ittel (Eds.), *Differenzierung im mathematisch-naturwissenschaftlichen Unterricht. Implikationen für Theorie und Praxis* (pp. 115–146). Bad Heilbrunn: Klinkhardt.

Schrader, F.-W. (2010). Diagnostische Kompetenz von Eltern und Lehrern [Diagnostic competence of parents and teachers]. In D. H. Rost (Ed.), *Handwörterbuch Pädagogische Psychologie* (4th ed., pp. 102–108). Weinheim: Beltz.

Schrader, F.-W. (2013). Diagnostische Kompetenz von Lehrpersonen [Diagnostic competence of teachers]. *Beiträge zur Lehrerbildung*, *31*(2), 154–165.

Schrader, F.-W., & Helmke, A. (2001). Alltägliche Leistungsbeurteilung durch Lehrer [Daily assessment through teachers]. In F. E. Weinert (Ed.), *Beltz-Pädagogik. Leistungsmessungen in Schulen* (pp. 45–58). Weinheim: Beltz.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, *12*(2), 151–169. doi:10.1007/BF00305619

van Es, E. A., & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education*, *10*(4), 571–596.

Weinert, F. E. (2000). *Lehren und Lernen für die Zukunft - Ansprüche an das Lernen in der Schule* [Teaching and learning for the future]. Pädagogisches Institut Bad Kreuznach.

Wiliam, D. & Thompson, M. (2007). Integrating assessment with instruction: What will it take to make it work? In C. A. Dwyer (Hrsg.), *The Future of Assessment. Shaping Teaching and Learning* (pp. 55–82). Mahwah, NJ: Routledge.