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GUIDED INQUIRY LEARNING OF FRACTIONS – A REPRESENTATIONAL APPROACH

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We present a theoretical model of a representational approach to inquiry based learning (IBL) in this paper. In IBL-environments, students investigate a mathematical domain by using multiple representations such as dynamic simulations and hands-on material guided by specially designed textbooks. In the empirical part, we describe a study focussing on self-generated representations by students with the aim of representing procedures and results.

Keywords: Self-generated representations, protocols, guided inquiry learning, fractions

CONCEPTUAL FRAMEWORK

In order to introduce the representational approach to inquiry based learning (IBL) in mathematics we define the concept of representational competence as two aspects that are two sides of the same coin. One side is the ability to manipulate and interpret prescribed external representations (representational input) and the other side is the ability to generate own external representations (representational output) (e.g. Izsák 2011, Schnotz, Baadte, Müller, Rasch 2011, Cox 1999). To build the link between IBL and representational competence, we need to define IBL:

> "Inquiry or scientific discovery learning environments are environments in which a domain is not directly offered to learners but in which learners have to induce the domain by experiments or examples." (de Jong 2005, p.215)

To specify this definition of IBL to inquiry in mathematics, we need to clarify what makes the work of a mathematician.

> “Mathematics proper might be regarded as the science of significant structure. Thus mathematics studies the representation of one structure by another, and much of the actual work of mathematics is to determine exactly what structure is preserved in that representation.“ (Kaput 1987 p.23)

Combining the essence of these two quotes it becomes clear that in IBL-environments in mathematics the goal for students should be to investigate a domain by analysing structures of given representations through examples or experiments with e.g. hands-on usable material or dynamic representations. This represents the first side of representational competence, the processing of representational input.

In IBL-environments the processing of the representational input in the cognitive system of each participating students is mediated by (1) social interaction within the group the students are working in, (2) interaction of the group with the learning environment or (3) personal interaction of an individual with the learning environment. After the processing in the individual cognitive system of each student, the students are supposed to generate representational output, which is also mediated by social or personal interaction. Our approach is in line with Tytler, Prain, Hubber and Waldrip (2013, p. 3) who see the need for the development of and research on IBL-environments in science learning *"with a strong explicit emphasis on student-generated representational work".* One major goal of generating representations during IBL is to represent results and solution steps externally. When it comes to student-generated representational work with the aim of presenting results and solution steps, we have to introduce the term "protocol". A protocol can be defined as a record, notation or description of essential stages phases and products of a learning process (e.g. an IBL-process) by using external representations such as texts, other symbols or diagrams (Dörfler 2000, p. 111f).

The aim of those protocols is to help the students to reflect on the IBL-process. By presenting results in form of protocols, these protocols become part of the learning environment and can therefore be part of the (social or personal) interaction with the learning environment in later stages of the inquiry-process. Furthermore, students can revise their protocols repeatedly during the inquiry process. What we described so far is our theoretical model of the representational approach to IBL (see Fig. 1). We have derived this model from classic input-output oriented information-processing models. In our model, the information-processing system is the cognitive system of the participating student. For the presented research, we consider the cognitive system (see Fig. 1) and therefore mental processes as a "black box". To get deeper knowledge on IBL from a representational point of view, we might have to open this "black box" in further research. For example the "integrated model of text and picture comprehension" of Schnotz (e.g. Schnotz et. al 2011) or other theories of cognitive processing in multimedia-learning could be used to open this black box.

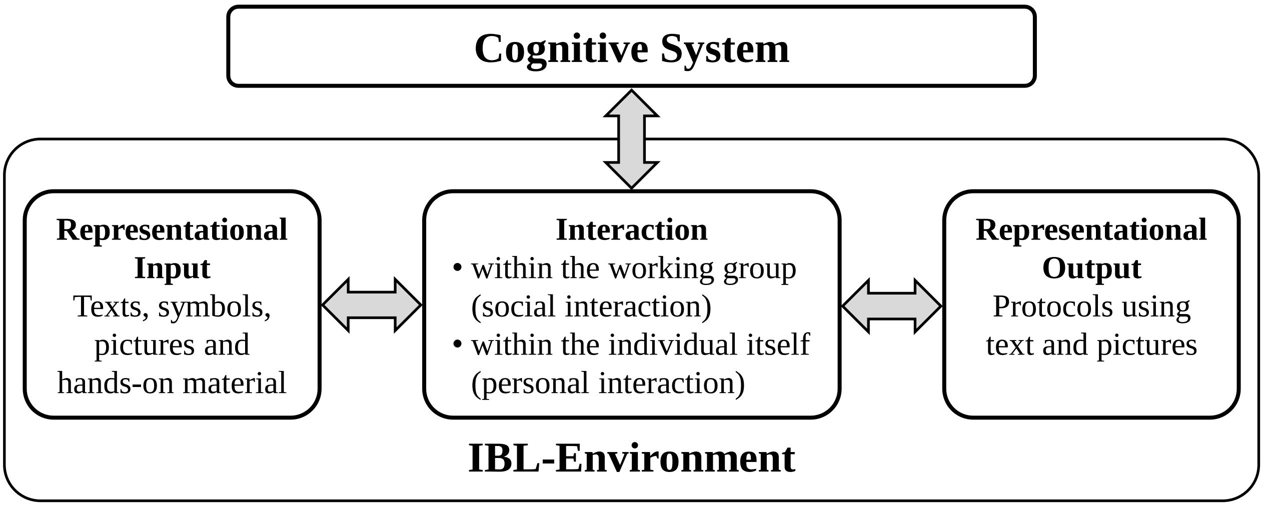


Fig. 1: Theoretical model of the representational approach to IBL

In this study, we focus on the individual student’s ability to generate protocols. We want to investigate

- how the individual’s ability to generate protocols (in the case of fractions) develops over time,
- in which way this development can be supported and
- how it is related to the individual’s content knowledge (in this case knowledge on fractions).

See the chapter “Research Design and Questions” for a detailed list of the investigated research questions. To investigate the research-questions we conducted a quasi-experimental intervention study with two IBL-conditions and one control condition. The focus of this article is to present the quantitative data from three measurement occasions (pre-, post-, follow-up test). We do not focus on the qualitative analysis of the interaction in the IBL-conditions, even though we videotaped some of the groups during IBL. Nevertheless we want to introduce the IBL-environment we used in the study to make the research more clear.

THE IBL-ENVIRONMENT

In the IBL-environment of the presented study, the students discovered fractions by analysing artworks of Max Bill who is one of the most famous representatives of the so-called “concrete art”. Max Bills “progression in five squares” (see the left side of Fig. 2) is one of the artworks we used. In this artwork, Max Bill arranged five equal squares in a column and split them progressively into smaller, but in each square equal sized rectangles. Because of the described structure, the artwork is suitable for students to explore the underlying structures of the “part-whole-concept of fractions”.

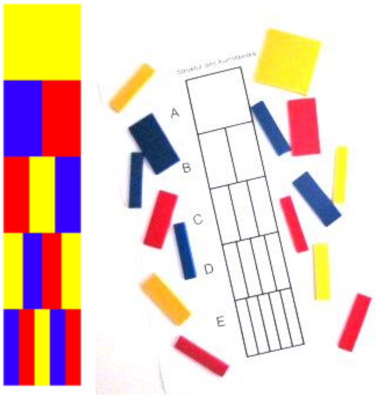
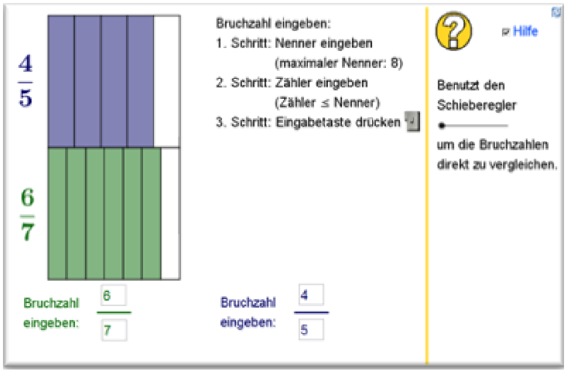


Fig. 2: Max Bill “progression in five squares” (left) and an equivalent fraction puzzle (right)

For every artwork presented in the IBL-environment hands-on material based on this artwork (see the right side of Fig. 2) was available for the students. It consists of the artworks outline structure on a laminated template and puzzle pieces of each coloured sub-area of the artwork. One can interpret each puzzle piece as a fraction of the whole artwork (or in the case of Fig. 2 as a fraction of one of the five squares). Therefore, we named those hands-on materials “fraction puzzles”. In Fig. 3 you can see a student using the fraction-puzzles to reason on an argument on the comparison of unit fractions, the working group discovered in their IBL-process. The students argument was that one third has to be “bigger” than one fourth, since in the case of the third the whole (the square) gets split into three parts, while in the case of the fourth the whole gets split into four parts. After writing down this argument in form of a protocol, the student in the green sweatshirt took a one half puzzle piece and two one fourth pieces (see Fig. 3) and said that now the whole also is split into three parts as well. A discussion started after that and the students decided to revise their so far produced protocol. They added that the parts the whole is split into have to be of equal seize.

Fig. 3: Students during an IBL-process

In addition to the fraction puzzles, students also had the opportunity to use dynamic visualizations (constructed using GeoGebra), again with a structure based on the artworks. Students can use these dynamic visualizations to test hypothesis they put up while dealing with hands-on material by further examples. Using the dynamic representation presented in Fig. 4 students can for example test hypothesis on different comparison strategies for fractions.



Bruchzahl eingeben:
1. Schritt: Nenner eingeben (maximaler Nenner: 8)
2. Schritt: Zähler eingeben (Zähler ≤ Nenner)
3. Schritt: Eingabetaste drücken

Benutzt den Schieberegler um die Bruchzahlen direkt zu vergleichen.

Bruchzahl eingeben: 6 / 7

Bruchzahl eingeben: 4 / 5

Fig 4. Screenshot of a dynamic visualization to compare fractions

Since guided IBL can be considered more successful than unguided IBL, we chose to implement a “triple support scheme” (De Jong 2005; Reid Zhan and Chen 2003) in the textbooks of our IBL-environment. We implemented *interpretative support* to help the students to interpret the prescribed representations. Whenever we present a task in the textbook that we anticipated as probably hard to solve for at least some of the students, a question mark icon ? indicates that there is some help provided in a special textbook. In this textbook, we present additional questions, hints or at the most solution steps but never the solution itself. We carried out *experimental support* to guide students while setting up experiments. This for example could be hints how to use the fraction-puzzles or the dynamic visualizations. In the presented study, we focus on *reflective support* with the aim of supporting students to generate protocols. We consider generating protocols as reflection, since students need to reflect on their learning process to be able to generate protocols. Prompts seem to be a promising approach to support the generation of protocols (Rau, Aleven, Rummel 2009, Berthold, Eysink, Renkl 2009; Berthold, Nückles, Renkl 2004). They are defined as “requests that require the learners to process the to be-learned contents in a specific way” (Berthold, Eysink, Renkl 2009). An example for a prompt we used is the following: “Represent the result of the task you just solved and reason why your result is correct using a sketch and a text." In our learning environment, we provide requests like this next to a framed space in the textbook in which the students can represent their results bearing in mind the prompt. When the framed space next to the prompt is empty, we consider this as *low instructional level*. In contrast to that, we consider to some extent pre-filled framed spaces as *higher instructional level*. In pre-filled framed spaces, we provide for example the beginning of a sentence or a first rough outline of a sketch, which the students have to accomplish in order to represent their results.

RESEARCH DESIGN AND QUESTIONS

To investigate the students ability to generate protocols in more detail we conducted a quasi-experimental study with two experimental IBL-conditions comparing two different instruction levels of reflective support through prompts to a control-group. The control-group was taught in a teacher-centred setting. The content of these teacher-centred lessons was the same as in the IBL-environment and in all three conditions students learned in three 90-minute lessons (see Table 1).

First Unit	• (Unit) fractions in the meaning of the part-whole concept • Comparing Fractions using meaningful semantic strategies on the basis of pictorial representations
Second Unit	• Repetition: Unit fractions in the meaning of the part-whole concept • Equivalence of fractions using graphical representations • Problematization of adding fractions on a semantic level
Third Unit	• Adding fractions on a semantic level through pictorial representations • Fractions with a value greater than one using the part-whole concept • Application of the reached results on a realistic problem situation

Table 1: Content of the learning unit

We kept learning-time and content consistent over all three conditions. Experimental-Group 1 (EG1) learned with textbooks using prompts on a higher instructional level. Experimental-Group 2 (EG2) learned with textbooks providing prompts on a low instructional level as described earlier.

In this research, eight sixth grade classes from two different German grammar schools took part. We randomly choose one class from each school as the control-group (CG). The other three classes of each school learned in the IBL-environment. The students in the IBL-environment learned together in groups of three to four students. We randomly assigned the students of each class to these working-groups and then each working-group randomly to one of the experimental conditions, by either providing them with textbooks containing prompts on higher (EG1) or low instructional level (EG2). By this distribution a total of $N = 81$ students were assigned to EG1, a total of $N = 68$ students were assigned to EG2 and a total of $N = 50$ students were assigned to CG, which means a total of $N = 199$ students took part in the study (including later dropouts).

We carried out the study in a pre-, post-, follow-up-test design and collected data on two variables at each of the three measurement-occasions. First data on the students' knowledge on fractions was collected and in a second step the students' ability to generate protocols was measured. For the measurement on fraction-knowledge a paper and pencil test was developed. This test has a special focus on the part-whole concept and operations amongst fractions (based on this concept) that were part of the intervention. The items in the test all focus on some kind of switch between representations. A typical task is to find the right pictorial representation for a given fraction in a multiple-choice-item.

For measuring the ability to generate protocols, we developed a new instrument based on so-called "video items". The underlying idea behind video items is to present a short video to the students during the test situation. This video shows a complete problem-solving process simulating IBL. In the case of the presented study, we used videos demonstrating a problem solving process on fractions, using hands-on materials like the fraction puzzles described earlier. For a screenshot of one the videos, see Fig. 5. By using videos, we want to simulate an IBL-process, which is in line with our theoretical model of the representational approach to IBL.

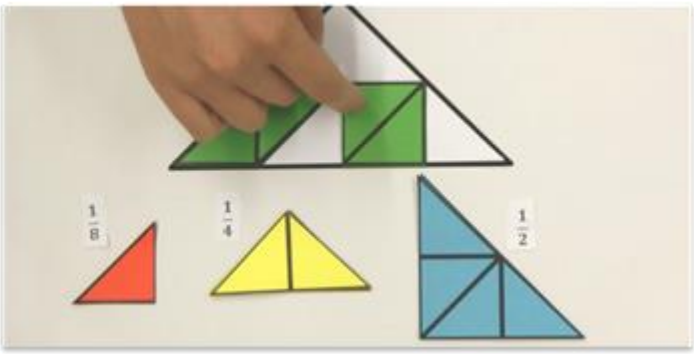


Fig. 5: Screenshot of a video item

The task for students in the test situation is to generate a protocol of the video-content using pen and paper after watching the video.

Raters can evaluate these protocols using categories like the correctness and completeness of the *represented contents* and of the *represented relations between the contents*. In Engl et al. (2014) we describe the concept of using video items to measure the ability to generate protocols in detail. See Fig. 6 for an example of a protocol based on a video-item.

To close this chapter we list the detailed research questions that contribute to the overall research-question we want to answer with this design:

Research question 1:
Do students in an IBL-setting achieve at least the same learning success as students in a teacher-centred setting?

Research question 2:
In which way does the ability to generate protocols develop over time under the three different conditions and is it possible to identify differences between the groups?

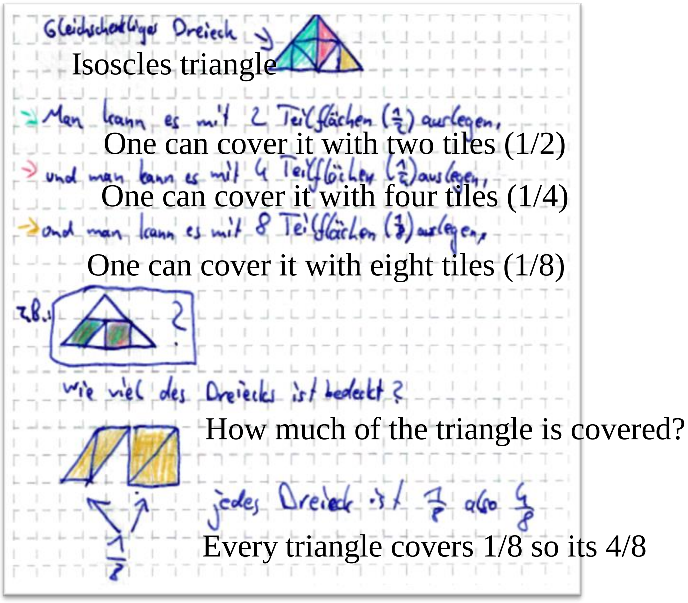


Fig. 6: Student generated protocol on the basis of a video-item

Research question 3:
Is there a correlation between the ability to generate protocols and the knowledge on fractions?

In the following, we present and discuss the results in detail. As said we put a special focus on the ability to generate protocols.

RESULTS

To investigate research question 1 we conducted a repeated measures ANOVA on the fractions-test scores, comparing the three conditions. The main-effect shows a highly significant increase in fraction knowledge over time for the three groups ($F(2,336) = 443{,}793$; $p < .01$). However no significant difference on the interaction between time and group could be detected ($F(4,336) = .986$; $p = .415$). This result shows that not only the teacher-centered setting can be considered successful. It indicates that the used support strategies for the IBL-environment lead to satisfactory learning-outcomes. With a focus on the treatment condition (reflective support through prompts), we can conclude that according to knowledge acquisition it makes no difference whether students are prompted on a high or low instructional-level in the case of the presented IBL-environment. This leads to the conclusion, that other design-principles (experimental and interpretative support) have more impact on knowledge acquisition than the different treatment conditions. What it does not mean is that students should not be prompted to generate protocols at all. We can say that independent of the instructional level of prompts, students learn successfully in the IBL-environment with the implemented support.

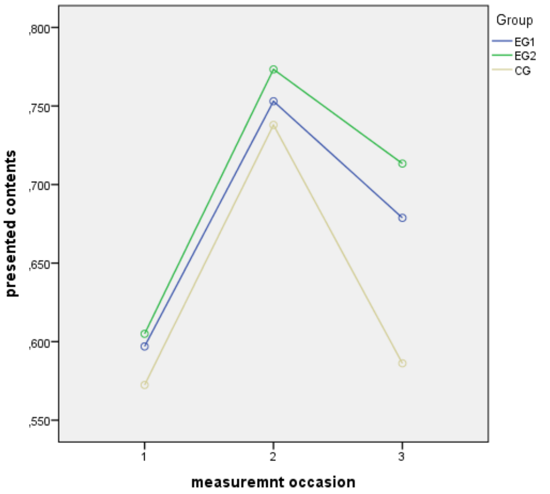


Fig. 7: relative values of presented contents in the protocols

Let us turn to the results relating the ability to generate protocols now. According to the main-effect, the repeated measures ANOVA reveals a highly significant increase in the numbers of correctly presented contents over time ($F(2,306) = 37{,}282$; $p < .01$). Looking at the interaction between group and time the repeated measures ANOVA does not reveal significant differences between the groups. ($F(4,304) = 1{,}142$; $p = .337$). However, a Tuckey-HSD Post-Hoc-Test detects a significant difference between EG2 and CG ($p < 0.05$).[1] This result becomes clearer when looking at the difference between EG2 and CG at the third measurement occasion in Fig. 7. The interpretation of this result is that EG2 shows a more sustainable ability to generate protocols than the CG. This is interesting because we provided EG2 with prompts on a low instructional level. EG1 also seems to show a more sustainable ability to generate protocols in comparison to CG, even though we cannot detect a significant difference.

What we can conclude from the significant main effect of the ANOVA on the generated protocols is that students develop the ability to represent results by learning on the topic, whether it is in an IBL-environment or a teacher-centered setting. However, students learning in the IBL-environment with prompts on a lower instructional level show a more sustainable ability to represent results.

Pre-Test	$r(180) = .217$; $p < .01$
Post-Test	$r(187) = .210$; $p < .01$
Follow-Up-Test	$r(167) = .224$; $p < .01$

Table 2: Pearson Correlations

Regarding research question 3, we detected low correlations between content knowledge and the ability to generate protocols at each measurement occasion (see Table 2). There are high achieving students regarding to fraction knowledge who fail to generate protocols and the other way around. This leads to the conclusion that content knowledge and the ability to generate protocols are two different constructs. We will discuss this interesting result amongst the other results in the closing section of this article.

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[1] Since the Tuckey-HSD Post-Hoc-Test uses pairwise testing, we can apply it without the ANOVA showing significant results (Hsu 1996 p.175f.).

CONCLUSIONS AND PRACTICAL IMPLICATIONS

As shown in the results section students improve their ability to generate protocols significantly over time. However, the ability to generate protocols is more sustainable for students who learned in the IBL condition. As for the students of EG2, who were guided to generate protocols by prompts on low instructional level (request to represent results next to empty framed spaces), the described effect was significant. Therefore, when it comes to reflective support, we recommend a low instructional level of prompts. Here we provided the prompts next to empty framed spaces. This means students are open to generate their own creative protocols, considering the hints how to generate the protocol given in the prompts. The benefit of the lower instructional level is that right or wrong, the students generate these protocols truly on their own and therefore represent their way of thinking. Therefore, the teacher can use them to get information on the insights the students reached. This would not be possible with pre-filled framed spaces, like in the first research-condition. When interpreting protocols we have to take into account that the quality of a generated protocol is not to be mixed up with high content knowledge. The low correlations between these two constructs clearly indicate this. To find out more about why the correlations are low, it might be interesting to open the black box and try to gain insight into students' mental models and the way they use them to generate protocols.

Another open research question is, whether students we consider "good representers" due to the results of the video items, really use their skills in IBL-settings and if not, how we have to design prompts to make them use their skills. Concerning this, it might also be interesting which factors influence the use of such skills. Motivation might have a huge impact, since our experience from watching students in IBL-environments indicates that the motivation to generate protocols is generally very low.

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