

# DEALING WITH COVARIATION: MISCONCEPTIONS AND THE EFFECT OF REPRESENTATION FORMS

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When dealing with the concept of function, secondary school teaching mainly focuses on the input-output view. In this view, every value of the domain corresponds to a precise value of the range. Researchers stress that a second aspect – the aspect of covariation – is not sufficiently implemented in mathematical curricula (e.g., Thompson, 1994). Covariational thinking requires taking the variation of input and output into account: In which way will the function value vary if the x-value is varied?

Two main research questions are:

- What kind of misconceptions surface when dealing with covariational tasks?
- Does the representation of the function (table of values vs. graph) have an effect on students' covariational thinking performance?

The sample comprised 27 students of grade 7. Using physical materials to build different models of cubes, the students explored and compared the covariation of linear and quadratic functions with a discrete domain within a 40-minute, guided intervention. For each function they were asked to produce a table of values and a graph as forms of representation. After the intervention, the students' covariational thinking performance was tested with six paper-and-pencil tasks.

In the qualitative analysis of misconceptions we discovered students' confusion between the first and second difference. Many students incorrectly identified a function with constant second differences as a linear function instead of a quadratic function. The quantitative analysis revealed that the form of representation had a large effect on the construction task. The students performed significantly better at value table production than graph construction (sign test:  $g = .35$ ,  $p < .001$ ). In the presentation, these and further findings will be discussed in detail.

These results raise the question of whether the table of values is a neglected form of representation for covariational reasoning since mathematics teaching mainly uses the graphical and symbolic approach. Furthermore, we conjecture that a table of values could be more adequate for quantitative exploration whereas a graph might be more appropriate for qualitatively exploring covariation. A further topic for investigation is the problem of how to avoid confusion between the first and second difference.

## References

Thompson, P. W. (1994). Students, Functions, and the Undergraduate Curriculum. In E. Dubinsky, A. H. Schoenfeld, & J. J. Kaput (Eds.), *Research in Collegiate Mathematics Education I* (pp. 21–44). Providence, RI: American Mathematical Society.